LETTERS TO THE EDITOR

To the Editor:

In the May 1983 *AIChE Journal* (p. 688), has suggested an alternative to the triregional model developed by Cohen and Metzner (1981) to account for the wall effect in the flow of Newtonian fluids through packed columns. The implications of the alternate two- and three-region models developed by Nield (1983) have not been fully explored in his original note. In this letter, we present a reevaluation of the model presented by Nield (1983).

The new feature in the model put forward by Nield (1983) was the treatment of the region in the immediate vicinity of the wall as a totally permeable annular ring (i.e., porosity of unity). It was further assumed, as suggested earlier by Beaver and Joseph (1967), that a slip boundary condition exists at the boundary between the porous medium and the annular wall layer. According to the two-region model, the volumetric flow rate Q, through a packed column, can be expressed by:

$$Q/Q_c = F_d + F_w \qquad (1)$$

in which Q_c is the flow rate, in a packed column with a radius R_c, calculated according to Darcy's law:

$$Q_c = \pi R_c^2 k \Delta P / L\mu \qquad (2)$$

where $\Delta P/L$ is the axial pressure gradient. F_d and F_w are the flow rates relative to Q_c, through the porous core region (of a uniform permeability), and the annular wall region (porosity of unity), respectively. F_d and F_w can be defined as,

$$F_d = n^2 \qquad (3)$$

and

$$F_w = \frac{1}{2}(1 - n_w)^2 + \frac{A[\frac{1}{2}(n^2 - 1)^2 - n^2 ln(n)]}{4\sigma^2} \qquad (4)$$

The variable n is the dimensionless radius of the porous core defined as $n = 1 - \gamma\sigma$, where $\sigma = \sqrt{k}/R_c$, and γ is the dimensionless thickness of the wall region, defined as $\tau = X_w/\sqrt{k}$. Finally, the constant A in Eq. 4 is given by the following function:

$$A = \frac{\Gamma n(1 - n^2) - 8n + 2n^2}{1 - \Gamma n\, ln(n)} \qquad (5)$$

in which $\Gamma = \alpha/\sigma$ and $B = 4\alpha\sigma$, where α is a dimensionless slip coefficient. The slip coefficient, α has been previously determined by Joseph and Beaver (1967) to range from a value of 4 at a permeability of 8.21×10^{-4} cm^2 to a constant value of 0.1 at a permeability less than 1.6×10^{-5} cm^2.

Nield (1983) has simplified Eqs. 1–5 by expanding in powers of σ and dropping terms of fourth order and higher (Eqs. 11–13 in the original paper). It was then argued that, according to the approximate equations, the ratio Q/Q_c is always less than unity, and thus the wall layer always leads to a reduction in the total flux for a given pressure gradient. It appears that this conclusion is unjustified since terms that are in fact nonnegligible have been dropped in the analysis. Therefore, to precisely evaluate the ratio Q/Q_c, Eq. 1 is reexamined numerically. First, however, to present the solution in terms of more convenient parameters, we introduce the hydraulic permeability (Dullien, 1975), $k = D_p^2\Phi^2$, in

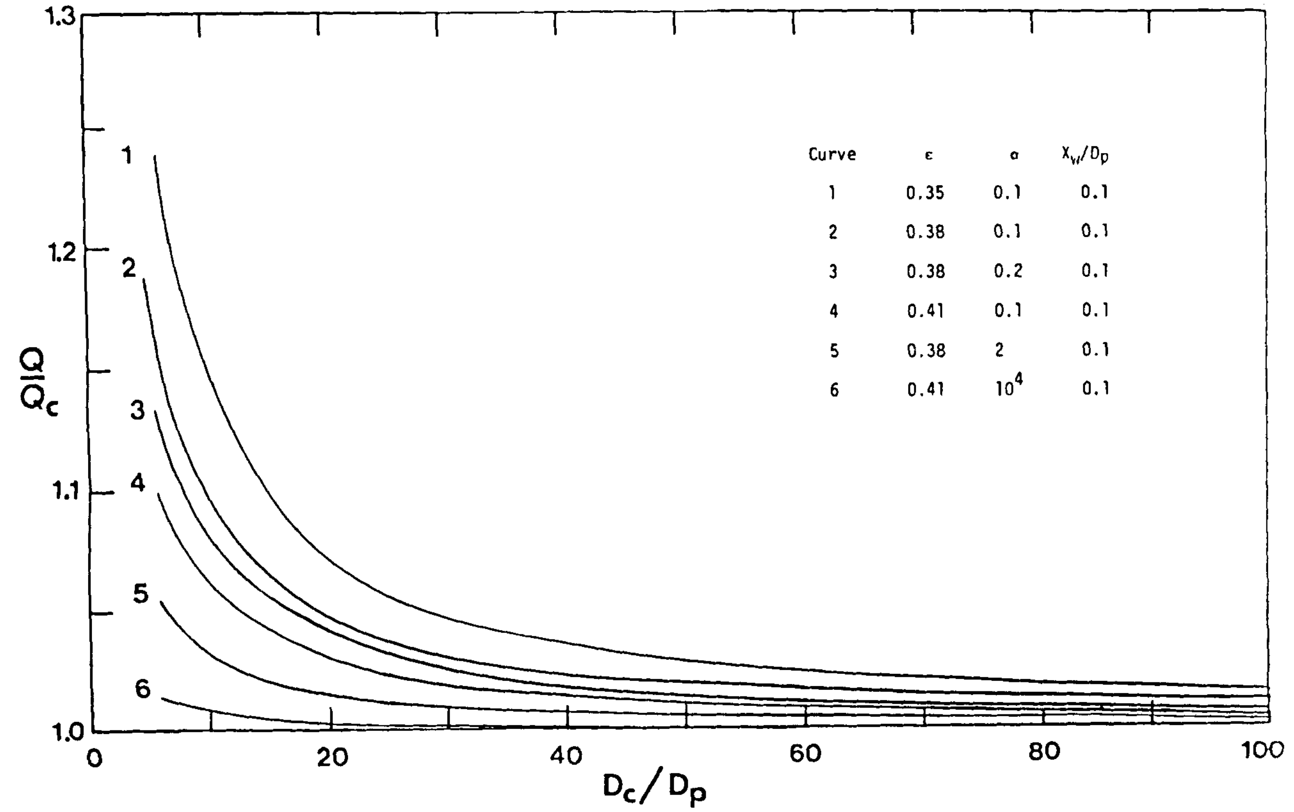

Figure 1. A comparison between the flow rate predicted by the two-region model and Darcy's law vs. D_c/D_p.

which $\Phi^2 = \epsilon^2/180(1-\epsilon)^2$. Therefore, the parameters in Eqs. 1–5 become: $\Gamma = \alpha m/2\Phi$, $B = 8\alpha\Phi/m$, and $\tau = 2(X_w/D_p)/\sigma m$, where m is the column particle diameter ratio (D_c/D_p). The predictions of Eq. 1 for various values of the core region porosity, the relative thickness of the annular wall region, X_w/D_p, and the slip coefficient α are shown in Figure 1. It is clearly demonstrated by the numerical results that the ratio of the fluxes Q/Q_c is greater than unity for small values of m, and it decreases toward the limit of unity as m increases. Q/Q_c is greater than unity even for very large values of the slip coefficient, α.

It is concluded that the two-region model of Nield (1983) predicts that the presence of a totally permeable wall region will always lead to a flow enhancement, relative to the Darcy's law prediction, at a given pressure drop. This is in contrast to the conclusion of Nield (1983) who suggested that the presence of the wall leads to a flow retardation. The same misinterpretation regarding the effect of the totally permeable annular wall region also exists in the three-region model proposed by Nield (1983). Contrary to the claim made by Nield (1983), his model is not in agreement with the model of Cohen and Metzner (1981).

NOTATION

A	= constant defined in Eq. 5
D_c	= diameter of packed column
D_p	= particle diameter
F_d, F_w	= flow rates in the porous core and wall region relative to Darcy's law flow for the column evaluated at the core porosity
k	= permeability of the porous core
L	= length of packed column
m	= column to particle diameter ratio
n	= dimensionless radius of core region
ΔP	= pressure drop across the packed column
Q	= flow rate through the column calculated from the two-region model
Q_c	= flow rate through the column calculated from Darcy's law using the core porosity
R_c	= column radius
X_w	= thickness of a wall layer of unit porosity

Greek Letters

α	= nondimensional slip coefficient
β	= $4\alpha\sigma$
ϵ	= porosity of the core region
μ	= fluid viscosity
Γ	= α/σ
Φ	= $[\epsilon^2/180(1-\epsilon)^2]^{1/2}$
γ	= $X_w/\sqrt{k}$, dimensionless thickness of wall region

Literature Cited

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Reply:

I am grateful to Dr. Cohen for pointing out that I made an algebraic slip in my paper (Nield, 1983a) discussing the wall effect in laminar flow of a fluid through a packed column. It turns out that the necessary correction improves the explanatory power of my two-region model.

In the above paper, Eq. 12 should read

$$F(\alpha,\gamma) = \frac{12 + 6\alpha\gamma - 4\gamma^2 - \alpha\gamma^3}{6 + 6\alpha\gamma}, \quad (1)$$

while Eq. 13 still stands:

$$Q/Q_0 = 1 - (d_f/R_c)F(\alpha, d_f/k^{1/2}). \quad (2)$$

where Q is the volume flux; Q_0 is the volume flux produced by the same pressure gradient in the absence of the wall effect; R_c is the radius of cross section of the circular cylinder forming the column; and $d_f = \gamma k^{1/2}$ is the thickness of the pure fluid layer at the wall (where γ is a numerical factor which is approximately unity), and α is the Beavers-Joseph constant.

We now observe that $F > 0$, and hence $Q < Q_0$, for all values of α provided that $\gamma < 3^{1/2}$. Thus, our model predicts the result that, even with a wall layer of porosity unity, a sufficiently thin wall layer has a restrictive effect, leading to a *reduction* in flux for a given pressure gradient. On the other hand, $F < 0$, and hence $Q > Q_0$, for all values of α provided that $\gamma > 6^{1/2}$. Thus, a sufficiently thick pure fluid layer leads to an *increase* in flux (as one might expect). Thus, our two-region model can explain either increase or reduction in flux, in accordance with the theoretical work of Cohen and Metzner (1981) and the experimental work discussed by them. There is no need to introduce a third region.

One can also attempt to model the wall effect using the Brinkman equation (for example, Nield, 1983b). A boundary layer whose thickness is of order $k^{1/2}$ arises at a wall when this equation is used. For the present situation, the Brinkman equation can be written

$$\frac{\Delta p}{L} + \mu\left(\frac{d^2U}{dR^2} + \frac{1}{R}\frac{dU}{dR}\right) - \frac{\mu U}{k} = 0. \quad (3)$$

This may be solved for the fluid velocity U subject to the no-slip condition

$$U = 0 \quad \text{at } R = R_c \quad (4)$$

and the requirement that U is finite at $R = 0$. The solution is:

$$U = \frac{k\Delta p}{L}\left\{1 - \frac{I_0(R/k^{1/2})}{I_0(R_c/k^{1/2})}\right\}. \quad (5)$$

where I_0 is a modified Bessel function of order zero. It has the asymptotic approximation

$$I_0(z) \sim (2\pi z)^{-1/2}e^z, \quad \text{as } |z| \to \infty.$$

Since $R_c/k^{1/2}$ will normally be large, we have approximately

$$U = (k\Delta p/L)\{1 - (R_c/R)^{1/2} \times \exp[-(R_c - R)/k^{1/2}]\}. \quad (6)$$

We see that in the bulk of the cylinder, $U = k\Delta p/L$, the Darcy value, but U is reduced to lower values in a boundary layer at the wall. It follows that the volume flux is reduced below the Darcy value for the flux. Thus, the Brinkman model predicts only a restrictive effect, and so is less useful than our two-region model.

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